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The Spin Structure of the Nucleon

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Abstract

We provide an introduction to the ideas of spin-dependent deep-inelastic scattering. Recent experimental results are summarised and possible explanations related to the axial anomaly presented. Further experiments that could greatly clarify the situation are also described.

Polarisation data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.

J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hadronic Processes, St. Croix, Virgin Islands (1987).

1   Introduction

Unravelling the mystery of the structure of the nucleon is one of the great challenges facing modern physics. On the theoretical side there are a vast array of QCD motivated models as well as the more fundamental lattice calculations. These are commonly compared with low energy data such as excitation energies and electoweak form-factors. In this regard it is somewhat surprising that little use has been made of deep-inelastic scattering data.

Deep-inelastic scattering (DIS) experiments provide perhaps our cleanest window on hadron structure at large momentum transfer squared Q^2. The original DIS experiments at SLAC in the 1960s showed that the form factors in ep scattering exhibit approximate *scaling* at large Q^2. This remarkable observation was celebrated recently when the 1990 Nobel Prize in Physics was awarded to the experimenters Friedman, Kendall and Taylor [1]. It gave rise to the original parton ideas of Feynman [2] and Bjorken and Paschos [3,4]. Precise data revealed the logarithmic scaling violations and also the gluon distribution that were predicted by QCD.

While the QCD improved parton model has proven extremely effective in correlating DIS data, it has only recently been taken seriously as a means to test and refine the models mentioned earlier [5, 6]. The motivation for doing so has increased dramatically since the measurement of the spin structure function $g_1^p(x)$ by the European Muon Collaboration (EMC) in 1988 [7]. That data has been quite widely interpreted as sparking a crisis – the "spin crisis".

Our purpose here is to introduce the basic ideas of the QCD improved parton model as it relates specifically to spin-dependent measurements. We shall review the current experimental situation and why the data was viewed as such a challenge to the quark model. We then discuss the axial anomaly in QCD and explain how it may contribute to polarised DIS.

This review is aimed at the broad audience of experimentalists and theorists not directly working in this area. It should be particularly valuable for graduate students interested in the problem. For those who want to follow the theoretical developments in more detail we refer to our recent review [8], as well as to the many original papers cited here. Other reviews with a different emphasis may be found in the lecture by Veneziano [9] and in the proceedings of the recent international conference on particles and nuclei [10]. Those for whom the experimental details are of greatest interest should refer to the recent review of Windmolders [11].

2   Deep Inelastic Scattering

Our primary concern is with deep-inelastic scattering from polarised targets – at CERN using muons (by the European Muon Collaboration (EMC) and the Spin Muon Collaboration (SMC)) and at SLAC using electrons. Consider a DIS experiment where a muon beam with definite polarisation and momentum $k^\mu = (E; \vec{k})$ scatters from a polarised proton target. At leading order in the electromagnetic interaction this is shown in the one photon exchange diagram of Fig. 1. We work in the laboratory frame so that the proton target has momentum $p^\mu = (m_p; 0_T, 0)$ and polarisation S^μ. The muon is scattered through an angle θ to emerge with momentum $k'^\mu = (E'; \vec{k'})$. The exchanged photon carries momentum $q^\mu = (k - k')^\mu$. The scattering process is then characterised by the two invariants $Q^2 = -q^2$ and $\nu = p.q$ ($\nu = m_p(E - E')$ in the LAB frame) or, equivalently, by Q^2 and the Bjorken variable $x = \frac{Q^2}{2\nu}$. We measure the inclusive hadronic cross section, so that hadronic final states, X, with the same invariant mass squared, $W^2 = (p + q)^2$, are not separated.

The differential cross section for the one photon exchange process (figure 1) is given by [12]

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}. \tag{1}$$

Here, α is the fine structure constant and $L_{\mu\nu}$ and $W_{\mu\nu}$ describe the muon and hadronic vertices respectively. Since the muon is elementary we can write down an exact expression for $L_{\mu\nu}$ from the Feynman rules, viz.

$$L_{\mu\nu} = 2\left[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k.k' - m_\mu^2) + i\epsilon_{\mu\nu\rho\sigma}q^\rho s^\sigma\right], \tag{2}$$

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where we take $\epsilon_{0123} = +1$. The hadronic tensor, $W^{\mu\nu}$, contains all of the information about the hadronic target that one can extract from such inclusive measurements. Its form is constrained by symmetry arguments. We write $W_{\mu\nu}$ as a sum of symmetric and antisymmetric contributions; viz. $W_{\mu\nu} = W_{\mu\nu}^S + iW_{\mu\nu}^A$. Then the requirements of covariance, parity, charge conjugation, and current conservation $(q^\mu W_{\mu\nu} = 0)$ imply the form

$$W_{\mu\nu}^S = \frac{1}{m_p}(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})F_1(x, Q^2) + \frac{1}{m_p\nu}(p_\mu - \frac{p.q}{q^2}q_\mu)(p_\nu - \frac{p.q}{q^2}q_\nu)F_2(x, Q^2) \tag{3}$$

and

$$W_{\mu\nu}^A = \frac{\epsilon_{\mu\nu\lambda\sigma}q^\lambda S^\sigma}{\nu}g_1(x, Q^2) + \frac{\epsilon_{\mu\nu\lambda\sigma}q^\lambda(\nu S^\sigma - q.S p^\sigma)}{\nu^2}g_2(x, Q^2) \tag{4}$$

The form factors in equs.(3) and (4) contain all of the target dependent information.

Deep inelastic scattering is defined by the kinematic limit where both $Q^2 \gg m_p^2$ and $W^2 \gg m_p^2$, so that we are beyond the resonance region (where W^2 may coincide with the mass squared of one of the excited nucleon resonances). In the DIS limit the form factors F_i and g_i in $W_{\mu\nu}$ exhibit approximate scaling. That is, they behave as structure functions of the single variable x - modulo a slow logarithmic variation in Q^2, which is described by perturbative QCD. The *scaling* property reveals a local interaction between the hard photon and charged elementary partons (quarks) inside the proton. It is the same effect as in Rutherford's α particle scattering experiments which revealed the nucleus inside the atom [13] - only at a much deeper level inside the nuclear "onion" [14].

We now turn to the measurement of the spin dependent structure function, g_1. This experiment involves a muon (or electron) beam and a proton target, both with longitudinal polarisation. We let $\uparrow\downarrow$ denote the longitudinal beam polarisation and $\Uparrow\Downarrow$ denote (longitudinal) polarisation of the target proton. Then the differential cross sections are

$$\frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE'} + \frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE'} = \frac{8\alpha^2(E')^2}{m_p Q^4}\left[2\sin^2\frac{\theta}{2} F_1(x, Q^2) + \frac{m_p^2}{\nu}\cos^2\frac{\theta}{2} F_2(x, Q^2)\right] \tag{5}$$

and

$$\frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE'} - \frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE'} = \frac{4\alpha^2 E'}{Q^2 E\nu}\left[(E + E'\cos\theta) g_1(x, Q^2) - 2xm_p g_2(x, Q^2)\right] \tag{6}$$

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One needs both a polarised beam and a polarised target to measure g_1. The structure function g_2 is suppressed with respect to g_1 by a factor $\frac{m_p}{E} \sim 0.01$, for a typical (muon) beam energy of 100GeV, and is lost among the errors. In this article we shall be interested only in g_1. Readers interested in the physics of g_2 should consult the review by Jaffe [15].

3   The parton model and sum-rules

The parton model began with Feynman who showed that the early SLAC deep inelastic scattering experiments could be explained in terms of the hard photon scattering incoherently from elementary *parton* constituents in the proton. The structure functions measure the probability for finding a quark with momentum fraction $x = \frac{p_+^{parton}}{p_+^{proton}}$ in the proton and which is polarised either in the same or the opposite direction to the proton's polarisation. This is usually called the naive parton model; it has no gauge degrees of freedom.

In the naive parton model the structure functions are described by the four linearly independent parton distributions. There are the two spin independent distributions. The charge conjugation even combination

$$(q + \overline{q})^{\uparrow}(x) + (q + \overline{q})^{\downarrow}(x) \tag{7}$$

occurs in the structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$. It is measured with an unpolarised target and either an electron or muon beam. In the naive parton model one finds

$$F_2(x) = 2xF_1(x) = x \sum_q e_q^2 \Big[(q + \overline{q})^{\uparrow}(x) + (q + \overline{q})^{\downarrow}(x) \Big], \tag{8}$$

where the relationship between F_1 and F_2 is called the Callan-Gross relation.

If the exchanged boson is a $W^{\pm}$ or a Z^0 instead of a photon then there is also a parity odd, C-odd term in $W_{\mu\nu}$. One finds a new structure function F_3, which may be measured with an (anti-)neutrino beam and a fixed target or with an electron beam at HERA, where there is a large centre of mass energy. The structure function $F_3^{\nu+\overline{\nu}}(x, Q^2)$ measures the C-odd valence distribution

$$(q - \overline{q})^{\uparrow}(x) + (q - \overline{q})^{\downarrow}(x) \tag{9}$$

The structure functions contain all the information about the target in DIS. It follows that when we take a given moment of a structure function we project out the target matrix element of some gauge-invariant, local operator. Indeed, this idea is formalised in the operator product expansion analysis of deep inelastic scattering. For example, the first moment of the unpolarised structure function F_2 (which is related to the the target matrix element of the energy momentum tensor) determines the fraction of the target's momentum carried by the quarks. Similarly, the first moment of the valence distribution tells us the number of valence quarks in the proton: three! (This is the Gross Llewellyn Smith sum rule which has been verified within experimental error.)

The moments of structure functions provide a testing ground for our understanding of proton structure in QCD. The first piece of experimental evidence that we needed to go beyond the naive parton model came when it was observed that quarks only contribute 50% of the proton's momentum. This was naturally accomodated with the advent of QCD where the gluons carry the momentum fraction

$$< xg(x, Q^2) > \rightarrow \frac{16}{16 + 3N_f}, \tag{10}$$

in the limit of infinite Q^2 [16]. (Here N_f is the number of quark flavours.)

Deep inelastic scattering is continuing to teach us about the QCD structure of hadrons. In the last decade we have seen an extensive programme to measure the structure functions of nuclear targets. This work was motivated by the discovery of the EMC effect ,namely that the structure functions of a nucleon are modified when the nucleon is in a nuclear environment [17, 18]. More recently, the unpolarised experiments at HERA are providing new information about the small x region and, in particular, the nature of the pomeron. The HERA experiments and E-665 at FNAL are also teaching us about the jet structure of deep inelastic scattering.

Whilst the physics of unpolarised deep inelastic scattering is reaching maturity, we are also seeing a whole new area of physics opening up - the spin dependent world! It is more than ten years since the first polarised deep inelastic scattering experiments were done at SLAC [19]. This work became the subject of renewed interest 5 years ago when EMC [7, 20] extended the SLAC measurements to smaller x and announced their data on the spin dependent proton structure function g_1. The naive parton model interpretation

of their data is that the quark partons contribute very little to the proton's spin - in contradiction with quark models.

During the last year we been treated to new spin data: the measurements of the deuteron structure function g_1^d by SMC at CERN [21] and the ^3He structure function by E-142 at SLAC [22]. From each of these measurements one can extract the neutron spin structure function g_1^n. Before we discuss the EMC experiment and the new data, let us review what the naive parton model says about g_1. This way we shall understand why the EMC result created so much interest.

In the naive parton model g_1 is written as:

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x) \tag{11}$$

where

$$\Delta q(x) = (q^\uparrow + \overline{q}^\uparrow)(x) - (q^\downarrow + \overline{q}^\downarrow)(x) \tag{12}$$

is the polarised quark distribution and e_q denotes the quark charge. It is helpful to rewrite $g_1(x)$ in terms of the SU(3) flavour combinations: $\Delta u(x) - \Delta d(x)$, $\Delta u(x) + \Delta d(x) - 2\Delta s(x)$ and $\Delta u(x) + \Delta d(x) + \Delta s(x)$.

In the naive parton model,

$$\Delta q = \int_0^1 dx \ \Delta q(x) \tag{13}$$

determines the fraction of the proton's spin which is carried by quarks (and anti-quarks) of flavour q. Thus, we write

$$\int_0^1 dx \ g_1(x) = \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{2}{9}\Delta c \tag{14}$$

where the singlet term $\Delta u + \Delta d + \Delta s$ denotes the light-quark spin content of the proton. (Here, we have included a contribution from charm quarks Δc, which is present above the charm threshold.)

In operator language, Δq is defined by the proton matrix element of the axial currents. We write

$$2m_p S_\mu \Delta q_a =< p, S|\overline{q}\gamma_\mu\gamma_5\frac{\lambda^a}{2}q|p, S > \tag{15}$$

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for $a = 3, 8, 0$. The non-singlet matrix elements also arise in the neutron and hyperon beta decays. Current algebra relates the spin dependent (strong interaction) structure of the proton measured im polarised deep inelastic scattering at high energies to the quantities needed in low energy weak interaction physics. The currents which measure Δq_3 and Δq_8 are soft operators so that these quantities are scale independent. They are determined as $\Delta q_3 = g_A = F + D$ and $\Delta q_8 = \frac{1}{\sqrt{3}}(3F - D)$ within SU(3). Here F and D are the antisymmetric and symmetric SU(3) couplings. One finds $\Delta u - \Delta d = 1.254 \pm 0.006$ and $\Delta u + \Delta d - 2\Delta s = 0.688 \pm 0.035$ [23] independent of scale. Since Δq_3 and Δq_8 are determined from other experiments we can extract the singlet "spin content" of the proton $\Delta u + \Delta d + \Delta s$. (This assumes that there is a negligible charm component Δc in the data.)

Before we can extract the "spin content" from g_1 data we also need to allow for the perturbative QCD Wilson coefficients. Ignoring charm, one finds:

$$\int_0^1 dx\, g_1^p(x, Q^2) = \frac{1}{6}\left[(\Delta q_3 + \frac{1}{\sqrt{3}}\Delta q_8)(1 - \frac{\alpha_s}{\pi}) + 2\sqrt{\frac{2}{3}}\Delta q_0(1 - \frac{\alpha_s}{\pi}\frac{33 - 8N_f}{33 - 2N_f})\right]$$
(16)

where the perturbative coefficients are quoted to $O(\alpha_s)$ [24]. In QCD the singlet term $\Delta q_0 = \frac{1}{2}\sqrt{\frac{2}{3}}(\Delta u + \Delta d + \Delta s)$ has an anomalous dimension, due to the axial anomaly, which starts at two loops [25].

There are two sum-rules for g_1, which have been tested in recent data. The Bjorken sum-rule [3] gives a relation for the difference between the first moment of g_1 for a proton and neutron target. To order α_s^3 precision it reads:

$$\int_0^1 dx\left(g_1^p(x) - g_1^n(x)\right) = \frac{1}{6}g_A^3\left[1 - \frac{\alpha_s}{\pi} + C_2(\frac{\alpha_s}{\pi})^2 + C_3(\frac{\alpha_s}{\pi})^3\right],$$
(17)

where $C_2 = -3.5833$ and $C_3 = -20.2153$ in the three flavour theory [26]. The Bjorken sum-rule was derived using current algebra before the advent of QCD and is a test of isospin.

The Ellis-Jaffe sum-rule [27] is a test of Zweig's rule (or OZI after Okuba, Zweig and Iizuka) in the flavour singlet channel. If we assume that strange (and heavy) quarks do not play a significant role and set $\Delta s = 0$, then the quark "spin content" would be determined by the hyperon beta decays. In this scenario we would have $\Delta q_0 = 0.69 \pm 0.03$, where the rest of the proton's spin would be carried by the gluons and also by quark and gluon

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orbital angular momentum. Substituting this number into equ.(16) we find $\int_0^1 dx \ g_1(x) = 0.189 \pm 0.005$ (at $10 GeV^2$). The Ellis-Jaffe sum-rule involves a model dependent assumption of good OZI , whereas the Bjorken sum-rule should hold exactly in QCD.

We now discuss the experimental results in polarised deep inelastic scattering and see how these sum-rules hold up.

4   Gifts from experiment

The European Muon Collaboration (EMC) measured the asymmetry

$$A(x, Q^2) = \frac{\frac{d^2\sigma \uparrow \Uparrow}{d\Omega dE'} - \frac{d^2\sigma \uparrow \Downarrow}{d\Omega dE'}}{\frac{d^2\sigma \uparrow \Uparrow}{d\Omega dE'} + \frac{d^2\sigma \uparrow \Downarrow}{d\Omega dE'}} \tag{18}$$

in the kinematic range $0.01 < x < 0.7$ and 1.5 GeV$^2 < Q^2 < 70$ GeV^2, with a mean Q^2 of 10.7 GeV^2 [7]. They used a polarised muon beam, which was generated by the decay of pions and kaons produced by primary protons taken from the CERN S$p\overline{p}$S. Muon beam energies of 100 GeV, 120 GeV and 200 GeV were used in the experiment. The target consisted of two sections polarised in opposite directions. Each was exposed simultaneously to the beam to reduce systematic errors. The beam polarisation was calculated as $(82 \pm 6)\%$ and the target polarisation was typically 75-80%. The data on $A(x, Q^2)$ was combined with measurements of the unpolarised structure functions to determine $g_1(x)$.

EMC found no significant Q^2 dependence in their own x bins nor when they compared with the previous experiments from SLAC, which took data in the range x between 0.18 and 0.70 and Q^2 between 3.5 GeV^2 and 10 GeV^2. Hence, they combined the two pieces of data assuming no Q^2 variation between them. The g_1^p data from EMC and SLAC is shown in Fig. 2. A smooth Regge extrapolation [28] ($g_1 \sim x^{-0.12}$ as $x \to 0$) was made to small x.

Thus, EMC found

$$\int_0^1 dx g_1(x) = 0.126 \pm 0.010 (stat.) \pm 0.015 (syst.), \tag{19}$$

which corresponds to the quark "spin content"

$$\Delta u + \Delta d + \Delta s = 0.120 \pm 0.094 (stat.) \pm 0.138 (syst.) \tag{20}$$

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at $< Q^2 >= 10.7 \text{GeV}^2$. This result is consistent with zero and two standard deviations from the Ellis-Jaffe hypothesis, which says that strange quarks should not play a significant role. For the various flavours, one finds $\Delta u = 0.78$, $\Delta d = -0.47$ and $\Delta s = -0.19$. In Fig. 3 we show the convergence of the first moment $\int_{x_m}^1 dx \ g_1(x)$ as a function of x_m. The large errors on the small x data points in the EMC g_1^p data do not have a significant impact on the integral.

This EMC spin effect (or proton "spin crisis") has inspired a large amount of theoretical work aimed at understanding the spin structure of the proton. It has also been the genesis of a new experimental program in polarised DIS.

One of the experimental priorities is to reduce the errors on the small x data points. Close and Roberts [29] suggested caution in adopting the Regge extrapolation at $x \sim 0.02$ and also examined the effect of SU(3) flavour violations on the F/D ratio and the subsequent extraction of the "spin content". It is clearly important to have accurate small x data, although it is not clear that one can resolve a problem with an operator sum-rule with data in the region $\ln \frac{1}{x} \gg \ln Q^2$, where the operator product expansion analysis is not applicable. The Spin Muon Collaboration (SMC) at CERN will take precise small x data with a polarised proton target in 1994. The HERMES experiment at HERA [30] will work at the same x range as EMC and should reduce the experimental error by a factor of 10.

The other experimental priority is to measure the spin dependent neutron structure function g_1^n and test the Bjorken sum-rule. This has been the focus of the two new pieces of data announced this year. The SMC [21] have measured g_1^d for a deuteron target in the kinematic range $0.006 < x < 0.6$ and 1 GeV^2 $< Q^2 <$ 30 GeV^2. They compare this data with the EMC proton data and this enables them (modulo small nuclear corrections) to extract information on the neutron structure function $g_1^n(x)$. SMC found

$$\int_0^1 dx(g_{1,EMC}^p - g_{1,SMC}^n) = 0.20 \pm 0.05(stat.) \pm 0.04(syst.), \qquad (21)$$

which agrees well with the theoretical Bjorken sum-rule prediction 0.191 ± 0.002, to $O(\alpha_s)$ at $Q^2 = 4.6$ GeV^2.

On the other hand, the E-142 experiment at SLAC [22] used a ^3He target to measure g_1^n. Their kinematic range was $0.03 < x < 0.6$ at an average $Q^2 = 2\text{GeV}^2$, which is lower than EMC and SMC. The polarised ^3He target is believed to act as a good model of the polarised neutron. E-142 used the

same analysis as we outlined in section 2 to test the Ellis-Jaffe sum-rule. They used the perturbative Wilson coefficients evaluated to order α_s and obtained

$$(\Delta u + \Delta d + \Delta s)_{SLAC} = 0.57 \pm 0.11, \tag{22}$$

corresponding to $\Delta s = -0.01 \pm 0.06$ at this Q^2, where $\alpha_s = 0.385 \pm 0.1$. This number is in agreement with the Ellis-Jaffe prediction! However, when E-142 combined their neutron data with the EMC proton data (evolved to $Q^2 = 2\text{GeV}^2$) they found

$$\int dx(g_{1,\ EMC}^p - g_{1,\ SLAC}^n) = 0.146 \pm 0.021. \tag{23}$$

This result is a two standard deviation discrepancy from the Bjorken sum-rule truncated to $O(\alpha_s)$ – which evaluates to 0.183 ± 0.007 at $Q^2 = 2$ GeV^2.

One might question the size of the error bar on equs.(22) and (23) since E-142 work only at $O(\alpha_s)$ and assume that the Bjorken sum-rule is exact at an intermediate stage of their analysis. (They assume isospin and use $\Delta u - \Delta d = g_A^3$ for the neutron in order to test the Ellis-Jaffe sum-rule. Hence, the errors on equs.(22) and (23) are not totally consistent.)

The apparent discrepancy between the two experiments done at different Q^2 suggests that we have to be careful about the Q^2 dependence of g_1. The higher order radiative corrections to the Bjorken sum-rule (to order α_s^3) which we quoted in equ.(17) were calculated by Larin, Gorishny and Vermaseren [26]. They are important at $Q^2 = 2\text{GeV}^2$, where the $O(\alpha_s^2)$ and $O(\alpha_s^3)$ corrections sum to give a nett correction, which is 60% of the size of the $O(\alpha_s)$ correction to the Bjorken sum-rule and each of these corrections carry the same negative sign. The theoretical prediction for the Bjorken sum-rule is 0.164 ± 0.009 at $O(\alpha_s^3)$ with $Q^2 = 2\text{GeV}^2$. This is clearly in agreement with equ.(23), within the experimental errors.

Ellis and Karliner [31] (see also Close and Roberts [32]) have pointed out that there may be higher twist effects present at the momentum transfer of the SLAC experiment and ask: "has scaling truely set in?" Nuclear corrections in ^3He [33] may also be important in understanding the SLAC data. On the other hand, it is interesting to note that the EMC and SMC data are taken above the charm threshold for much of their x range, whereas the E-142 experiment is consistently below charm threshold. Charm quarks turn on rapidly as one goes through threshold in the unpolarised structure function

[34]. It is tempting to think that the same is true with g_1 [35]. Brodsky has emphasised that there may be an intrinsic charm component in the proton at finite x and that this charm would be strongly polarised [36]. It would be interesting to measure J/Ψ production in polarised DIS following NMC [37] - only as a function of the target polarisation.

Our theoretical discussion to this point has focussed on g_1 in the naive parton model, which is pre-QCD. Just as the discovery that the quark partons only carry 50% of the proton's momentum led to the discovery of the gluon distribution so the EMC spin effect has led to a new understanding of the role of spin in QCD. There has been a considerable theoretical effort in the last five years trying to understand the violation of the Ellis-Jaffe sum-rule in the EMC data. Kodaira [25] had previously pointed to the role of the axial anomaly [38, 39] in polarised deep inelastic scattering. After the announcement of the EMC data there was a considerable theoretical debate [40-54] aimed at understanding the exact role of the anomaly and how it contributes to the first moment of g_1^p. More recently [55, 56], it has been shown that the anomaly is relevant to the whole spin dependent structure function. The EMC spin effect should be thought of as an all moment problem!

Unfortunately, much of the discussion of the anomaly in the literature has been rather formal. In the next section we shall endeavour to explain the idea behind its role in polarised deep-inelastic scattering in terms of simple quantum mechanics.

5 Theory: the structure of the proton

One of the main theoretical problems when analysing deep inelastic scattering and proton structure is to find a sensible way to break the proton up into quark and gluonic components. The spirit of QCD motivated models of the proton is that we consider the three valence quarks in some confining background field, which is associated with condensates in the vacuum. These potential models consist of a marriage of classical field theory and relativistic quantum mechanics, in the spirit of Dirac's hole theory.

To see how they compare with quantum field theory we follow ref. [57] (see also [58]) and consider the quarks in an external gluonic field. The traditional picture uses a Fock state expansion of the proton, where vac-

uum effects with momentum greater than some large ultraviolet cut-off λ are ignored. However, this recipe leads to an energy-momentum tensor, $\theta_{\mu\nu}$, which is not conserved at $O(\alpha_s)$. There is a nett flow of energy from the world of finite momenta over the λ cut-off in momentum space and into the vacuum [57]. The analogous situation in classical physics is a system in thermodynamic equilibrium with a large heat bath. There can be a nett flow of energy between the system and the heat bath with the system at equilibrium. Whilst the *bare* energy in this process is not conserved one can write down a *free* energy, which is conserved. In quantum field theory, the physical $\theta_{\mu\nu}$ is obtained by adding in a reciprocal *regulator* term, which represents a flux of energy and momentum from the vacuum back to the world of finite momenta. This physical $\theta_{\mu\nu}$ plays the role of the free energy in thermodynamics. However, there is a trade off: we obtain an effective conserved energy momentum tensor but this $\theta_{\mu\nu}$ is no longer traceless for massless quarks. One finds

$$\theta^\mu_\mu = (1 + \gamma_m) \sum_q m_q \overline{q} q + \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu} G^{\mu\nu}, \tag{24}$$

where m_q is the running quark mass and γ_m is the mass anomalous dimension, $G_{\mu\nu}$ is the gluon field tensor and $\beta(\alpha_s)$ is the QCD beta function. The gluonic term is the trace anomaly, which is very important in QCD.

Classical physics is scale invariant: there is no running coupling constant. This scale symmetry is manifest via Noether's theorem in terms of the partially conserved dilation current $\partial^\mu s_\mu = \theta^\mu_\mu$. The trace anomaly signifies a scale violation in quantum field theory. It is intimately related to the running of α_s and thereby to the scale dependence of the parton distributions, which are measured in deep inelastic scattering. Furthermore, the proton mass is determined by the trace anomaly [59, 46]. The matrix element of $\theta_{\mu\nu}$ in the proton is

$$< p|\theta_{\mu\nu}|p > = 2p_\mu p_\nu. \tag{25}$$

Taking the trace of this equation we find

$$< p|\theta^\mu_\mu|p > = 2m_p^2. \tag{26}$$

Clearly, gluons are important here: in a free quark model with no glue the "proton" would be three massless, unconfined quarks with total mass zero. In a semi-classical quark model (say the MIT bag model) this gluonic component

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appears as the infinite confining potential well in which the quarks live. When we say that the three valence quarks are at large x in DIS (viz. that they carry a lot of the proton's momentum) we should remember that this proton mass or momentum is generated via the trace anomaly. The valence quarks are observed to carry 39% of the proton's momentum at $Q^2 = 15\text{GeV}^2$ [18]. This means that in the limit of massless quarks

$$2m_p^2 \int_0^1 dx \; x(u_v + d_v)(x) = 0.39 < p, S|\frac{\beta(\alpha_s)}{4\alpha_s}G_{\mu\nu}G^{\mu\nu}|p, S > \qquad (27)$$

In this sense the structure function F_2 measures the distribution of the trace anomaly over x.

The flux of quanta between the quarks with finite momenta and the vacuum also induces a second effect (the axial anomaly), which is measured in polarised deep inelastic scattering. In quantum field theory the conserved axial vector current is not invariant under gauge transformations of the external field! The physical (gauge invariant) current differs from the conserved (free) current by a regulator term, which corresponds to a leakage of spin (or chirality) over the cut-off λ in momentum space to the background gluonic field [57, 59]. This is the axial anomaly. It induces a two loop anomalous dimension in the physical axial vector current, which means that this current does not measure the quark spin content of the proton.

In the naive parton model we can write down the spin operators for quarks in their rest frame [60]

$$S_k = \int d^3x \; (\overline{q}\gamma_k\gamma_5 q), \qquad k = 1, 2, 3. \qquad (28)$$

The S_k are time independent for massless quarks and satisfy the algebra of SU(2) spin. In QCD, the anomalous dimension means that the flavour singlet axial current at a scale Q^2 is

$$j_{\mu,5}^0(Q^2) = j_{\mu,5}^0(Q_0^2) \; Z_5(Q_0^2 \to Q^2), \qquad (29)$$

where Z_5 is the renormalisation group factor. Hence, even if the algebra of SU(2) spin is satisfied at some scale, Q_0^2, at Q^2 we find:

$$[S_i^{QCD}, S_j^{QCD}](Q^2) = i\epsilon_{ijk}S_k^{QCD}(Q_0^2) \; Z_5(Q_0^2 \to Q^2). \qquad (30)$$

Thus the S_k cannot satisfy SU(2) in QCD! We now discuss how the axial anomaly appears in g_1.

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6 g_1 in QCD

Deep inelastic scattering provides us with a microscope, which we use to probe the QCD structure of the proton. The resolution is determined by the momentum transfer squared Q^2 carried by the hard photon. In order to analyse DIS in QCD we use the operator product expansion and QCD evolution. The flavour singlet part of g_1 receives contributions from both quark and gluon partons, viz.

$$g_1(x, Q^2)|_S = \frac{1}{3}\sqrt{\frac{2}{3}} \int_x^1 \frac{dz}{z} \left[\Delta q_0(z, Q^2) C^q(\frac{x}{z}, \alpha_s) + \frac{1}{\sqrt{6}} \Delta g(z, Q^2) C^g(\frac{x}{z}, \alpha_s) \right]$$
(31)

The C-even, spin dependent quark $\Delta q(x, Q^2)$ and gluon $\Delta g(x, Q^2)$ distributions contain all the information about the target. They have the interpretation that they determine the probability for finding a quark or gluon respectively, which carries a fraction x of the plus component of the target's momentum (p_+). The coefficients $C(x, \alpha_s)$ are target independent and describe the interaction of the hard photon γ^* with each of these quark and gluon partons. We now explain the physics behind this formula.

The problem of seperating the proton into quark and gluonic components has two ingredients. We first need to make sure that our quark and gluon distributions have no infra-red divergences. Secondly, we have to renormalise the proton wavefunction in a manner consistent with the symmetries of QCD at strong coupling.

If we work in perturbation theory, then the first of these problems is solved by the factorisation theorem. Consider DIS from a quark or gluon target to some order in α_s. This cross section has logarithmic mass singularities, which arise from parton branching in the collinear direction (ie. at vanishing k_T^2). The mass singularities are an infra-red effect. They determine the logarithmic factors (anomalous dimensions) which govern the QCD evolution of the parton distributions. The great achievement of the parton idea was the proof by Amati, Petronzio, Veneziano and others [61] (see also the earlier work [62]) that these mass singularities factorise to all orders in perturbation theory. We can write the cross section for DIS from a quark or gluon target as the convolution of a term which contains all the mass singularities with a coefficient term which is "hard", by which we mean that it is infra-red safe. Since there are no free quarks and gluons in nature, the

confinement mechanism must somehow act to smooth out the infra-red mass singularities. The factorisation theorem allows us to write the structure function in the form of equ.(31), where we identify the infra-red term with the parton distributions.

Having taken care of the mass-singularities we then have to renormalise the proton wavefunction or, equivalently, the parton distributions in equ.(31). At this point it is helpful to consider the role of the uncertainty principle in DIS. This is important when the hard photon interacts with a gluonic component in the proton, where this interaction takes place entirely within one Compton wavelength of the photon. The photon can resolve only that it has made a pointlike interaction with an electrically charged parton within this distance. It cannot resolve whether it has interacted with a quark or with a gluonic component in the target.

A natural signal for the interaction with a gluon is the perturbative photon gluon fusion process, which generates two quark jets with large $k_T^2 \sim Q^2$ according to one's favourite jet algorithm [64]. When we calculate photon gluon fusion we start by treating the gluon as the target. Away from perturbation theory it is not clear (see below) that this approximation extends to when the hard photon makes a local interaction with the classical background glue, which is associated with the confining potential in QCD inspired models of the proton. This second process may have a different jet signature from the perturbative photon gluon fusion.

The uncertainty principle tells us that there is no unique separation of the local $\gamma^*(\text{QCD})$ interaction with the quarks or with the background field in which they sit. In unpolarised DIS the uncertainty principle is just the renormalisation scheme dependence of parton distributions. For example, we can renormalise the proton wavefunction using either the MS or $\overline{MS}$ schemes. Depending on how we have defined a "gluon", the local γ^*g interaction is shuffled between the coefficient $C^g(x, \alpha_s)$ (which measures the interaction of the photon with a gluon parton) and a local coupling (via $C^q(x, \alpha_s)$) to the quark distribution $(q + \overline{q})(x, Q^2)$ or proton wavefunction. (The parton model assumes that we can choose the coefficient to be equal to the hard photon-parton cross-section in the full theory – when we go beyond perturbation theory.) The structure functions F_1 and g_1 are measured with transverse photons, whereas F_2 receives contributions from both transverse and longitudinal photons. The polarisation direction of the photon sets the uncertainty in the local interaction. If the photon is longitudinally polarised

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the polarisation is orthogonal to the transverse plane and there is no uncertainty: the longitudinal coefficients C_L^q and C_L^g are scheme independent (see eg. the text by Roberts [65]).

At this point it is worthwhile to make a few remarks about the parton model in QCD, where the coefficients are taken to be the hard photon-parton cross-sections. In perturbation theory the mass singularities factorise universally to all high-energy, inclusive hadron scattering processes [62]. This result suggests that we may use the parton distributions measured in deep inelastic scattering experiments to predict cross-sections for other high-energy processes, which are given as the convolution of these parton distributions with the cross-sections for parton sub-processes leading to the requisite final state. This parton model hypothesis is exact in perturbation theory so any violation of it would be a signal of non-perturbative physics – and perhaps provide a window on the background field (or QCD vacuum) which leads to confinement. The parton model has been tested to $O(\alpha_s^3)$ precision in unpolarised $p\overline{p}$ scattering at CDF, where there is excellent agreement between theory and data [63]. However, as we explain below, the parton separation between hard and soft is less reliable in spin dependent processes.

In polarised DIS some of the uncertainty is frozen out by the gauge symmetry of QCD. The parton distributions (or proton wavefunction) in g_1 are defined by the operator product expansion so that their odd moments project out the target matrix elements of the *renormalised*, spin-odd, composite operators

$$2m_p S_+(p_+)^{2n} \int_0^1 dx\, x^{2n} \Delta q_k(x, Q^2) = \, <p, S|[\overline{q}(0)\gamma_+\gamma_5(iD_+)^{2n}\frac{\lambda^k}{2}q(0)]_{Q^2}^{GI}|p, S>_c$$

(32)

$$2m_p S_+(p_+)^{2n} \int_0^1 dx\, x^{2n} \Delta g(x, Q^2) = \, <p, S|[\text{Tr}\, G_{+\alpha}(0)(iD_+)^{2n-1}\tilde{G}_+^\alpha(0)]_{Q^2}^{GI}|p, S>_c \ (n \geq 1)$$

(33)

Here $D_\mu = \partial_\mu + igA_\mu$ is the gauge covariant derivative in QCD. The association of $\Delta q_k(x, Q^2)$ with quarks and $\Delta g(x, Q^2)$ with gluons follows when we evaluate the target matrix elements in equs.(32) and (33) in the light-cone gauge, where $D_+ \rightarrow \partial_+$. (The superscript on the operators emphasises that they are quoted with respect to some gauge invariant regularisation scheme, and the subscript Q^2 indicates the subtraction point.)

The parton distributions are renormalised quantities which should always be quoted with respect to some ultraviolet regularisation. As we explained

in section 5, the choice of regularisation determines the symmetry properties of the theory. The clash of symmetries between different regularisations is what is meant by an anomaly. (For a complete discussion of regulator theory we refer to the text of Zinn-Justin [66] and the lectures of Guichon [67].) The physical axial vector current does not measure a quark spin content because of its anomalous dimension. In classical physics the axial vector current is gauge invariant: the quark field transforms as

$$q(x) \rightarrow U(x)q(x)$$

and

$$\overline{q}(x)\gamma_\mu\gamma_5 \rightarrow \overline{q}(x)\gamma_\mu\gamma_5 U^\dagger(x) \tag{34}$$

under a given gauge transformation U. On the other hand, in quantum field theory the axial vector current operator is not just $\overline{q}(0)\gamma_\mu\gamma_5$ multiplied by $q(0)$. It is a composite operator which has to be renormalised and there are extra divergences which are intrinsic to the operator itself. The conserved axial vector current, which does measure spin, is the symmetry current [68]

$$j^S_{5\ \mu} = j^{GI}_{5\ \mu} - k_\mu, \tag{35}$$

Here

$$k_\mu = \frac{\alpha_s}{2\pi}\epsilon_{\mu\lambda\alpha\beta}\mathbf{Tr}\ A^\lambda(G^{\alpha\beta} - \frac{2}{3}A^\alpha A^\beta) \tag{36}$$

is a gauge dependent gluonic current, which measures the leakage of spin to the background field. The gauge dependent symmetry current $j^S_{5\mu}$ is *the* axial-vector current in a world where chirality is an exact symmetry and where gauge invariance is not. The anomaly in the axial vector current is important in QED, where it predicts the decay rate for $\pi^0 \rightarrow 2\gamma$ [38, 39]. This decay would be strongly suppressed without the anomaly. The axial anomaly is also relevant to our understanding of the U(1) problem in QCD [69].

It is important to realise that the anomaly is not restricted to an effect in the axial-vector current. An anomaly will manifest itself wherever the quantum numbers of the relevant operator allow it. For example, any renormalisation calculation involves a subtraction scale - and this breaks the scale invariance of the classical theory. The only operators which are scale invariant are those which are not renormalised (ie. conserved currents and the other "soft" operators - the divergences of which just involve mass terms).

The QCD evolution of the parton densities with increasing Q^2 and asymptotic freedom are intimately related to the scale anomaly. It turns out that one cannot renormalise each of the $C = +1$ axial tensor operators in a gauge invariant way so that they describe spin at the same time. The operator which does measure spin differs from the corresponding gauge invariant operator by a multiple of a gauge-dependent, gluonic counterterm $k_{\mu\mu_1...\mu_{2n}}$, viz.

$$\left[\overline{q}(0)\gamma_\mu\gamma_5 D_{\mu_1}...D_{\mu_{2n}}q(0)\right]^S_{Q^2} = \left[\overline{q}(0)\gamma_\mu\gamma_5 D_{\mu_1}...D_{\mu_{2n}}q(0)\right]^{GI}_{Q^2} + \lambda_{S,n}\left[k_{\mu\mu_1...\mu_{2n}}\right]_{Q^2} \tag{37}$$

where the coefficients $\lambda_{S,n}$ and anomalous currents were discussed in ref. [55].

The operator product expansion analysis of deep inelastic scattering involves only gauge-invariant *operators*, whence it follows that the spin dependent quark distribution in DIS does not measure the quark spin content of the proton. (Gauge invariance wins out over chirality!) Instead, it includes a local interaction between the hard photon and the background field. One can say that *the gauge symmetry screens the spin of the quarks.*

Since the only operator which contributes to the first moment of g_1 is the gauge-invariant axial-vector current and this operator is associated with the quark distribution $\Delta q(x, Q^2)$, it follows that

$$\int_0^1 dx \Delta g(x, Q^2) \int_0^1 dz C^g(z, \alpha_s) = 0 \tag{38}$$

In general, $\Delta g(x, Q^2)$ is target dependent, whence one concludes that the first moment of the spin dependent gluonic coefficient must vanish to all orders in perturbation theory at the twist-two level [55] (see also [48]).

At this point it is important to understand the physical implications of this discussion. In the parton model we would like to set the gluonic coefficient equal to the hard cross-section for photon-gluon fusion. This means including the local $\gamma^* g$ interaction, which is associated with the anomaly, into $C^g(x, \alpha_s)$, whereby the first moment of g_1 would measure the spin of the quarks minus a contribution from the polarised gluon distribution $\Delta g(x, Q^2)$, viz. $-\frac{\alpha_s}{2\pi}N_f\Delta g$ [41-44]. However, this corresponds to a parton model, where the quark distribution is defined with respect to the S-prescription axial tensor operators. (That is, it lives in the world where chirality is an exact symmetry at the expense of gauge-invariance.) It is well known from the

work of Jaffe and Manohar [46] that the forward matrix elements of k_μ (and by extension the higher spin operators, $k_{\mu\mu_1...\mu_{2n}}$) are not gauge-invariant beyond perturbation theory. In otherwords, the parton model corresponds to a regularisation which breaks the gauge symmetry at strong coupling when we consider spin-dependent processes.

Nachtmann and collaborators [71] have pointed out that the parton model hypothesis may also breakdown when one considers spin in the Drell-Yan process due to non-perturbative aspects of the background field inside hadrons. They point to evidence of this effect in the NA-10 data [72]. The parton model idea that we can make a clear separation of high-energy, hadronic scattering processes into hard and soft fails when we consider spin dependent processes.

7 Phenomenology of the anomaly in g_1

We now illustrate this discussion by comparing g_1 with the other structure functions, which are measured in DIS experiments. The axial anomaly is relevant only to g_1. It is not present in the unpolarised quark distributions, which are described in OPE language by the operators $\overline{q}(0)\gamma_+(iD_+)^n q(0)$. Nor is it relevant to the C-odd spin dependent structure function g_3, which is the polarised version of F_3. (In the naive parton model g_3 measures the spin dependent valence distribution $(q-\overline{q})^\uparrow(x) - (q-\overline{q})^\downarrow(x)$.) Since g_3 is odd under charge conjugation, it can have no anomalous gluonic contribution. (This result follows from the Furry theorem.) This means that it does make sense to talk about F_1, F_3 and g_3 in terms of quarks with explicit spin degrees of freedom - the clash of symmetry between gauge invariance and spin does not manifest itself in these structure functions.

In order to deal with g_1 we should modify the parton spin identification in equ. (12) by writing the gauge invariant distribution as [56, 73]

$$q_{GI}^\uparrow(x, Q^2) = (q_S^\uparrow + \frac{1}{4}\kappa)(x, Q^2) \qquad (39a)$$

$$q_{GI}^\downarrow(x, Q^2) = (q_S^\downarrow - \frac{1}{4}\kappa)(x, Q^2) \qquad (39b)$$

for both quarks and anti-quarks. Here $\kappa(x, Q^2)$ denotes the anomaly and the subscript S denotes the gauge-dependent spin distribution. The κ distribution appears only in the treatment of g_1 in deep inelastic scattering. Since

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the anomaly is independent of quark flavour the same κ distribution is relevant to each of $u, d, s, c, ...$ It varies only according to the "spin" and not the charge or flavour. Because κ is flavour independent it will induce some OZI violation wherever it plays a role. This is the likely source of the EMC spin effect.

If we substitute equs.(39) into the OPE expression for g_1 (viz. equ.(31)) it is easy to see that both the spin and anomalous components of $\Delta q(x, Q^2)$ couple to the hard photon in exactly the same way as one expects of a quark (via $C^q(x, \alpha_s)$). Physically, this means that there is a new local interaction between the hard photon and a gluonic component in the proton, which must be included in the parton model [54, 55] This is despite the fact that the glue does not carry electric charge !

We now discuss the renormalisation group properties of the anomaly in $g_1(x, Q^2)$. At $O(\alpha_s)$ the measured parton distributions evolve with increasing Q^2 according to the GLDAP equations [74]

$$\frac{d}{dt}\Delta q_0(x, t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[\Delta q_0(z, t)P_{qq}^q(\frac{x}{z}) + 2N_f \Delta g(z, t)P_{gg}^q(\frac{x}{z}) \right]$$

$$\frac{d}{dt}\Delta g(x, t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[\Delta q_0(z, t)P_{qq}^g(\frac{x}{z}) + \Delta g(z, t)P_{gg}^g(\frac{x}{z}) \right] \qquad (40)$$

where $t = \ln Q^2$. The splitting functions have a probability interpretation in the light-cone gauge so that eg. $P_{gg}^q(x)$ is the probability for a gluon to branch into a quark-antiquark pair so that the quark (or antiquark) carries a fraction x of the gluon's momentum in the collinear direction. The splitting function is the coefficient of $\ln \frac{Q^2}{\mu^2}$ in the cross section for DIS from a quark or gluon at $O(\alpha_s)$.

In section 6 we explained that the anomaly κ appears as a local interaction of the hard photon with a gluon, where the interaction takes place within one Compton wavelength of the photon. In perturbation theory, this corresponds to $k_T^2 \sim Q^2$ and there is no logarithm at $O(\alpha_s)$. This means that we can disentangle an evolution equation for κ [56] from the GLDAP equations:

$$\frac{d}{dt}\kappa(x, t) = 0. \qquad (41a)$$

All of the QCD evolution of $\Delta q(x, Q^2)$ at $O(\alpha_s)$ is carried by its spin component. If we substitute equs.(39) into equs.(40) we can re-write the GLDAP

equations as

$$\frac{d}{dt}\Delta g(x,t) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[(\Delta q_S(z,t)+\kappa(z,t))P_{qq}^g(\frac{x}{z})+\Delta g(z,t)P_{gg}^g(\frac{x}{z})\right] \quad (41b)$$

$$\frac{d}{dt}\Delta q_S(x,t) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[(\Delta q_S(z,t)+\kappa(z,t))P_{qq}^q(\frac{x}{z})+2N_f\Delta g(z,t)P_{gg}^q(\frac{x}{z})\right]$$
$$(41c)$$

The anomaly $\kappa(x,Q^2)$ scales at $O(\alpha_s)$ and has a different evolution equation from the partonic gluon distribution $\Delta g(x,Q^2)$. Clearly, the anomaly is an entirely different gluon effect in g_1 than $\Delta g(x,Q^2)$.

We can now write down the properties of the anomaly in g_1.

- The background field (or confining potential) in QCD is important if we wish to understand the shape of the structure functions in deep inelastic scattering. The trace and axial anomalies describe a flux of quanta between the quarks with finite momenta and the vacuum. The trace anomaly determines the proton mass in terms of the confining potential. The axial anomaly means that the spin of the quarks is screened by the background field in which they live.

- In general, the spin and anomalous components of $\Delta q(x,Q^2)$ are not separately gauge invariant. Rather than try to separate $\Delta q(x,Q^2)$ into meaningful quark and gluonic components we take the view that one should keep with the gauge invariant distributions in equ.(31), while bearing in mind that $\Delta q(x,Q^2)$ contains some OZI violation associated with the axial anomaly.

- The anomaly in $\Delta q(x,Q^2)$ scales at $O(\alpha_s)$ and couples to the hard photon in exactly the same way as a quark. Hence the anomaly makes a scaling contribution to g_1.

It is clearly an important experimental challenge to map out the x dependence of the OZI violation in g_1. In section 5 we explained how the unpolarised structure function can be viewed as the distribution of the trace anomaly over x: the trace anomaly is relevant to large x! Formally, it is clear that the axial anomaly is also important at large x since it is intrinsic to each of the higher moments of $\Delta q(x,Q^2)$. Since the anomaly is not

present in the spin-dependent valence distribution, g_1 and the valence part of g_1 probe different aspects of what we call the constituent quark. The interesting phenomenology of a large x anomaly is a possible violation of Zweig's rule in the large x bins of the polarised DIS experiment. Of course, the size of this OZI violation at large x is a problem for experiment. The point is that there is no good theoretical reason to expect that the anomaly is constrained to small x. A large x anomaly would mean that the spin of the three valence quarks is screened by the confining potential in g_1. It would be seen as a violation of Zweig's rule in the large x bins of the polarised DIS experiment. This would be in contrast with the ordinary gluon distribution $\Delta g(x, Q^2)$, which is relevant to g_1 only as small x ($x \leq 0.03$) [47, 49, 50].

To see that ultraviolet regularisation effects (anomalies) can be present at large x it is also helpful to consider the light-cone correlation function in polarised DIS. At the semi-classical level we can write $\Delta q(x, Q^2)$ as the Fourier transform

$$\Delta q(x, Q^2) = \frac{1}{2\pi} \int dz_- \cos(xp_+z_-) < p, S|\overline{q}(z_-)\gamma_+\gamma_5 q(0)|p, S >, \qquad (42)$$

where this object requires careful renormalisation in the full quantum field theory [75]. By inverting the Fourier transform for typical valence and sea distributions one finds [75] that the correlation function for the valence distribution is peaked about $z = 0$, whereas large correlation lengths ($z_- > 1$fm) are associated only with small x. Small z_- contributes to the full range of x.

At this stage we consider Schwinger's derivation of the anomaly [76] using point splitting regularisation. The physical axial vector current is obtained as the $z \rightarrow 0$ limit of

$$\overline{q}(z)\gamma_+\gamma_5 e^{ie \int A_\mu dy^\mu} q(0) \qquad (43)$$

acting in an external field. Modulo the path ordered exponential this is the same object that appears in the correlation function equ.(42). In the Schwinger derivation z acts as the ultraviolet regulator and the anomaly appears as a term $\sim \frac{1}{z}$ in the expansion of equ.(43) about $z = 0$. The anomaly is associated with small correlation lengths and therefore has the potential to contribute at large x.

8 Modelling the axial anomaly in g_1

Since the anomaly is not present in the valence (C-odd) part of g_1 it would be most helpful to obtain data on the C-odd spin dependent proton structure function. A large x anomaly would show up as a finite difference between g_1 and its valence part at large x [55, 73]. This difference should be associated with a second gluon distribution in polarised DIS, which is a clear window on the confining potential in the proton. If the anomaly is purely a small x effect it would not be possible to isolate it from the sea and the ordinary gluon distribution $\Delta g(x, Q^2)$, which dominate the small x data. (We shall come back to the ordinary gluon distribution at the end of this section.)

Unfortunately, the cross section for DIS with a neutrino beam and proton target is very small - enough to make direct measurements of g_3 impracticable at the present time. However, in the naive parton model it is possible to extract the C-odd valence distribution from g_1 measurements by detecting fast pions from among the final state hadrons [77, 78]. Whilst this analysis needs to be updated in the light of the anomaly it is clear that semi-inclusive measurements provide the best chance to measure the spin dependent valence distribution at the present time. This experiment is planned by the HERMES collaboration at HERA [30] – preliminary data is also available from SMC, albeit with large errors.

Important information about the x dependence of the axial anomaly in polarised deep inelastic scattering also comes from measurements of $g_1^n(x, Q^2)$. The axial anomaly occurs only in the flavour singlet part of g_1 and therefore it will be present equally in g_1^p and g_1^n as a function of x. If the anomaly acts to screen the quark spin at large x in g_1^p it follows that the same should be true in g_1^n. The combination appearing in the Bjorken sum-rule $(g_1^p - g_1^n)$ has no flavour singlet component and is anomaly free. On the other hand, the flavour singlet component is enhanced in the deuteron structure function $g_1^d = \frac{1}{2}(g_1^p + g_1^n)$, which has no isotriplet piece $\Delta q_3(x, Q^2)$. Thus the deuteron structure function g_1^d is an ideal place to test model predictions about how the anomaly should contribute in the nucleon structure function $g_1(x, Q^2)$.

The usual quark model calculations, which do not include the anomaly, suggest that g_1^n will change sign and become small and positive at large x [79-82]. To the extent that these models do not include any OZI violation, a large x anomaly would tend to render g_1^n negative at large x. As a specific example we consider the quark model calculation of the structure functions

which was developed by the Adelaide group [80-82] following earlier work by Jaffe, Ross and others [83]. These calculations use the MIT bag model, a popular, relativistic model of hadron structure, whch reproduces low energy hadronic properties such as charge radii, magnetic moments and the flavour non-singlet axial charges. This same model also provides a good description of unpolarised structure function data, notably $xF_3(x)$ and the d/u ratio. For polarised deep inelastic scattering the model also respects the Bjorken sumrule and, because it has no OZI violation, the Ellis-Jaffe sum-rule. (The bag model has not yet been extended to satisfy the U(1) chiral Ward identity.) Because of this last feature it is clear that it cannot fit the EMC data for g_1^p. Nevertheless, it does seem reasonable that the bag model prediction for g_3^p should be reliable since this structure function is free of any Zweig's rule violations due to the anomaly.

In view of the general success of the model in unpolarised physics and our earlier description of the role of the anomaly, which is certainly missing in the bag treatment, we make the working hypothesis that the discrepancy in g_1^p is entirely due to the anomaly. By adding a purely phenomenological term to the bag model calculations to fit the EMC data, we are then in a position to make predictions for g_1^n and g_1^d. This is illustrated in Fig. 4a. Figure 4a shows the EMC and earlier SLAC data for xg_1^p together with the naive bag model expectation (the solid line) corresponding to a bag radius $R = 0.8$fm, which we take from Schreiber et al. [81]. The dashed curve is the result of adding a purely phenomenological term to fit the data [84]. We will associate this phenomenological term with the OZI violation missing in the naive bag model. In this case, the same flavour singlet correction should therefore be applied to the neutron and this is shown in Fig. 4b. Here the solid curve is the naive bag (no OZI violation) prediction for xg_1^n (again at $Q^2 = 10\text{GeV}^2$ for a bag radius $R = 0.8$fm) and the dashed curve is obtained by adding the same correction that was applied for the proton. Clearly when this correction is included we find that the neutron spin-dependent structure function becomes negative at large x. In Fig. 4c we show the naive bag (solid) and anomaly corrected (dashed) predictions for xg_1^d. (For the present purposes we make the simple approximation that $g_1^d = (g_1^p + g_1^n)/2$,thus ignoring corrections due to shadowing,Fermi-motion and the D-state probability of the deuteron. These are expected to be important at the few-percent level [85] - well below the present experimental accuracy.) The corrected curve is in good agreement with the recent SMC measurement of the deuteron spin structure function

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$xg_1^d(x, Q^2)$ [21].

We repeat this analysis in Figs. 5a-5c for a bag radius of $R = 0.6$fm. The bag model calculation of g_1 is taken from ref.[79], while the dashed curve is again the bag result supplemented by a purely phenomenological term [86]. Again the bag model supplemented by the anomaly term tends to favour a negative sign for g_1^n at large x, and there is reasonable agreement with the SMC data for xg_1^d. If the anomaly is an intrinsic part of the constituent quark then one would expect it to have the same $(1-x)^3$ behaviour, which is given by the counting rules. It is interesting to note that the phenomenological term in our calculations, which we associate with the anomaly, roughly reproduces this shape in the middle x range where the data is most reliable.

We have stressed the difference between the anomaly and the ordinary gluon distribution $\Delta g(x, Q^2)$. It is known from unpolarised DIS experiments that the gluon distribution is concentrated at small x. In polarised DIS the hard photon scatters from the gluon distribution $\Delta g(x, Q^2)$ via a quark-antiquark pair, described in $C^g(x, \alpha_s)$, viz.

$$g_1^G(x, Q^2) = \frac{1}{9} \int_x^1 \frac{dz}{z} \Delta g(z, Q^2) C^g(\frac{x}{z}, \alpha_s) \qquad (44)$$

This dissipates the gluon's already small momentum so that $\Delta g(x, Q^2)$ is relevant to g_1 only at small x ($x \leq 0.03$) [49,50].

At present we do not have a measurement of $\Delta g(x, Q^2)$ so we need to deduce some phenomenological parametrisation to estimate this gluon contribution in the g_1 data. The hardest possible $\Delta g(x, Q^2)$ that we can write down consistent with the inequality

$$|\Delta g(x) = (g^\uparrow - g^\downarrow)(x)| \leq g(x) = (g^\uparrow + g^\downarrow)(x) \qquad (45)$$

is

$$\Delta g(x) = x^\alpha g(x) \qquad (46)$$

where $\alpha \geq 0$. For definiteness, consider the experimental parametrisation [84]

$$xg(x, Q^2) = 0.88(1 + 9x)(1 - x)^4 \qquad (47)$$

at $Q^2 = 4\text{GeV}^2$. (The EMC low x data was measured at $< Q^2 >= 4\text{GeV}^2$ so we can use equ.(46) to estimate the gluon component in this part of the data.) We use the $\overline{MS}$ coefficient $C^g(x, \alpha_s)$ [48, 53] and take $\alpha = 0.2$ in equ.(46),

26

which corresponds to a large polarised gluon component $\Delta g = 4.0$ in the proton. (A large $\Delta g \sim 4$ has been postulated by several authors [41-44].) The contribution of $\Delta g(x, Q^2)$ to the EMC data is shown in Fig. 6, where we plot the combination $(g_1^{EMCdata} + g_1^G)(x)$. This plot allows us to see the gluonic contribution to the EMC data on g_1 at a glance. The gluon distribution $\Delta g(x, Q^2)$ contributes to g_1^p only at small x and is clearly well within the experimental errors where data has been taken. It makes a negligible contribution to the measured sum rule between $x = 0.01$ and 1, where the three constituent quarks are expected to dominate. This result is essentially independent of the choice of the parametrisation for $\Delta g(x)$. (Similar calculations, using a variety of parametrisations for $\Delta g(x)$ have appeared in refs. [47,49-50,88].) Whilst the gluon distribution $\Delta g(x, Q^2)$ does not contribute to the first moment of g_1 (equ.(38)), the sign of Δg clearly affects the low x region ($x \leq 0.01$). *With present data even the sign of Δg is undetermined.*

In concluding this section we should record the fact that there have been a large number of papers devoted to the calculation of corrections to traditional models of nucleon structure that have the effect of correcting the Ellis-Jaffe sum-rule. The nearest in spirit to the work emphasised here is that involving perturbative corrections arising from instantons [89, 90] – since instantons break axial U(1) symmetry. Additional mechanisms considered have included gluonic and pionic corrections [91-94]. All of the latter shift the spin of the nucleon into orbital angular momentum of nucleon constituents (pions,quarks and gluons). It is a very important challenge to theory and experiment to understand quantitatively the role played by such corrections in comparison with the more fundamental explanation on which we have focussed.

9 Elastic neutrino proton scattering

There have been a large number of theoretical papers [46, 95-98]. suggesting that the value of Δs extracted from the EMC measurement of g_1 should be compared with a quantity (also denoted "Δs") which is measured in elastic neutrino proton scattering. This point needs some clarification.

Elastic neutrino proton scattering proceeds at leading order through the neutral current (single Z^0 exchange). In the notation of equ.(15), the Z^0 couples to the combination $\Delta u - \Delta d - \Delta s + \Delta c$, which is flavour non singlet; the νp cross section is anomaly free.

The term Δc is commonly omitted and this is the source of much confusion. An intrinsic part of the νp cross section involves the Z^0 coupling to the target via two gluons. This is shown in Fig. 7. It is necessary to include charm here because all flavours of quark can flow through the virtual loop in the vector (gluon), vector (gluon) axial-vector (Z^0) subgraph. It is not correct to ignore the charm quarks just because of the heavy quark mass. If the charm contribution is neglected we cannot satisfy current conservation at each of the Z^0 and gluon vertices. For a given flavour of quark, we can ensure current conservation at the gluon vertices via the anomaly. However, we cannot then renormalise the triangle amplitude to ensure current conservation at the Z^0 vertex. This would mean subtracting out the anomalous counterterm which we introduced to maintain gauge invariance at the gluon legs. The anomaly is flavour independent. Hence, there is no problem when we consider the full non singlet coupling at the Z^0 vertex : the nasty divergences which act to destroy gauge invariance cancel out and we have a sensible theory. (The apparent loss of gauge invariance at the Z^0 vertex due to the mass terms in the divergence equation is restored by the Higgs mechanism.) We must sum over the full generation of quarks to ensure current conservation and, hence, renormalisability. Indeed, this is the reason for demanding a top (truth) quark in the theory - to cancel the anomaly associated with the beautiful quark.

The value of "Δs" which is determined in elastic neutrino proton scattering is really $\Delta s - \Delta c$. This combination is scale independent. It should not be compared directly with the Δs that is extracted from the EMC experiment and which does have an anomaly and is scale dependent. The currently accepted value is $\Delta s - \Delta c = -0.15 \pm 0.08$ [99], which agrees within errors with the EMC value for $\Delta s = -0.19$. However, as the experimenters themselves point out [99] (see also Close [95]), the value of $\Delta s - \Delta c$ which is extracted from the neutrino experiments is strongly dependent upon the dipole mass term in the axial form factor. By varying this mass parameter within one standard deviation, one easily finds that $\Delta s - \Delta c$ is consistent with zero.

Although it may appear to belabour the point we feel that it is worthwhile to make one further point concerning neutrino proton (νp) scattering. The virtual Z^0 involved in this process carries some four momentum transfer Q^2, so that in general the matrix element of $\overline{q}\gamma_\mu\gamma_5 q$ is taken between proton states of different momentum. This introduces a form-factor which as we mentioned above is usually parameterised as a dipole (with mass parame-

ter about 1GeV). It would be incorrect to confuse this dependence with the scale dependence of quantities measured in DIS, where the forward Compton amplitude involves initial and final protons of the same momentum. In particular, it would not be correct to suggest that at (say) zero momentm transfer νp scattering would determine Δs at a scale $Q^2 = 0$. The combination which is actually measured, namely $\Delta s - \Delta c$, is scale invariant. By itself Δs has a scale dependence induced by the anomaly, but if Δs were to grow by an amount δ when changing scale from μ_0^2 to μ^2, then Δc would grow by the same amount.

The interpretation of the deep inelastic process cross section depends on whether we are above or below the threshold for the production of real charm. This differs from the elastic neutrino proton scattering where the charm quarks are purely virtual. Whether or not we can apply the OPE is determined by the kinematics of the physical process. By applying the OPE we are assuming that Q^2 is much greater than all the other QCD scales in the problem. This is fine for the light quarks. Far below charm threshold ($Q^2 \ll 10\text{GeV}^2$) charm quarks will be frozen out of DIS. Well above threshold ($Q^2 \gg 10\text{GeV}^2$) they participate fully and $\Delta c^{(DIS)}$ is given by relation to the OPE as the proton matrix element of the axial vector current $\overline{c}\gamma_\mu\gamma_5 c$,

$$2mS_\mu\Delta c^{(DIS)} = < p, S|\overline{c}\gamma_\mu\gamma_5 c|p, S > . \tag{48}$$

Near the threshold the problem is complicated - for a detailed discussion we refer to Witten [100]. In any case, the situations for DIS and νp elastic scattering (where the charm quarks are purely virtual) are physically quite distinct [35].

10   Conclusions and outlook

In this review we have outlined the experimental results for spin dependent deep inelastic scattering that have generated so much interest. The reasons for that interest in terms of the traditional parton model were then explained. Having established this background we were able to concentrate on the deeper theoretical issues. We made it clear that the EMC spin effect is *not* a question of the fraction of the spin of the nucleon carried by its quarks; when gauge invariance is respected this is not a well-defined concept.

Although attention has been traditionally focussed on the violation of the Ellis-Jaffe sum-rule – that is, the violation of OZI in the first moment of g_1 – we have been at pains to emphasise that this throws away a great deal of information. In particular, the conflict between chiral symmetry and gauge invariance that lies behind the U(1) axial anomaly occurs in every moment of g_1. Indeed we have argued that the spin structure of the nucleon offers a unique window into nucleon structure and the mechanism of confinement. At first glance this makes our approach quite different from the approach of Shore and Veneziano [9, 45] who argue that the OZI violation is target independent – being related to the scale dependence of the decay constant of the analogue of the η' in OZI QCD. The approach of ref.[45] and that discussed here should be related at some deeper level (as both approaches stem from the conflict between gauge invariance and chiral symmetry mentioned above). However, there remains an open problem of how to generalise their analysis to address higher moments (i.e. the *shape* of g_1).

We have discussed how one could map out the x dependence of the axial anomaly in g_1. If the anomaly does produce effects at large x it can be isolated as a finite difference between the structure functions g_1 and g_3 in the large x bins. If it is purely a small x effect it would be lost among the sea and gluon distributions which dominate the data in this region (say ≤ 0.1). Certainly, the anomaly is an intrinsic property of what we call the constituent quark and there is no good theoretical reason to believe that it is confined to small x.

We strongly urge that consideration be given to the challenging experimental problem of how to measure g_3. In the interim it would be very useful to obtain more data (with reduced errors) on the deuteron spin structure function g_1^d. The HERMES experiment aims to reduce the errors on g_1^d by a factor of 10 [30]. This new deuteron data will be able to discriminate between the two theoretical curves in Figs.4c and 5c and will provide an important constraint on models of nucleon structure. At the same time it is important to map out the Q^2-dependence of the data, especially through the charm threshold. Finally, little attention has so far been paid to the possible ambiguities in extracting g_1^n from experiments on nuclei where the nucleons are necessarily off-shell. It has recently been shown that this can lead to a substantial correction in the unpolarised case [101], and mindful of Bjorken's Law (quoted at the beginning) one must worry even more about the polarised case.

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<div align="center">**Figures**</div>

1. **Fig. 1: Deep Inelastic Scattering.**
   Here, we work in the LAB frame where the proton target (mass M) has momentum $p^\mu = (M; 0_T, 0)$ and polarisation S^μ. The incident muon (mass m) carries momentum $k^\mu = (E; \vec{k})$ and polarisation s^μ. It is scattered through an angle θ so that the scattered muon emerges with momentum $k'^\mu = (E'; \vec{k'})$. The exchanged photon carries momentum $q^\mu = (k - k')^\mu$. We measure the total inclusive cross section so that the final state hadrons X are not separated.

2. **Fig. 2:** The world data on $g_1^p(x)$ [7,19].

3. **Fig. 3:** The convergence of the first moment $\int_{x_m}^1 dx\, g_1(x)$ as a function of x_m in the EMC data [7].

4. **Figs. 4a-4c:** Fig.4a shows the world data for xg_1^p [7, 19] together with the bag model expectation [81] (solid curve), for a bag radius $R = 0.8$fm. The dashed curve is obtained by adding a phenomenological term to fit the data. Figs.4b and 4c show the naive bag (solid curve) and OZI corrected (dashed curve) prediction for xg_1^n and xg_1^d respectively, together with the SLAC neutron data [22] and the SMC data for xg_1^d [21].

5. **Figs. 5a-c:** Fig.5a shows the world data for xg_1^p [7,19] together with the bag model expectation [82] (solid line), for a bag radius $R = 0.6$fm. We fit the data by adding a phenomenological term (dashed curve). Figs.5b and 5c show the naive bag (solid curve) and OZI corrected (dashed curve) prediction for xg_1^n and xg_1^d respectively, together with the SLAC neutron data [22] and the SMC data for xg_1^d [21].

6. **Fig. 6:** The contribution to g_1^p from $\Delta g(x)$. We show the combination $(g_1^{EMCdata} + g_1^G)(x)$ (dashed curve) where $C^g(x, \alpha_s)$ is calculated in the $\overline{MS}$ scheme. We use the parametrisation equ.(44) with $\alpha = 0.2$ for $\Delta g(x)$. The bold curve is the EMC fit and small x extrapolation. Clearly, this gluon contribution is concentrated at small x - essentially outside the range of the present data.

<div align="center">39</div>

7. **Fig. 7:** Contribution to the neutrino proton cross section where the Z^0 couples to the proton via two gluons.